Related Rates and Optimization

By Mitchell Philipp and Nicolás F. Bartholomäi, with some guest problems by Powell Whitaker and Cody “Malfeasance” Nelson

Review - Related Rates

Organize the problem into “Givens”, “Find”, and “Relationship”. Don’t forget any “hidden givens.” Plug in any constants that you already know, and that never change. If it’s something like “find the rate of change of y WHEN x=3, you cannot plug in the x until after taking the derivative. Take the derivative of the relationship, and plug in all the givens. Then, just solve the derivative for whatever the “find” is. It may also be helpful to draw a diagram.

Review - Optimization

The basic gist here is to model the question as a function, and then find either the max or min, depending on the problem. There are three methods to solve for the max or min:

1. Take the derivative, find all the critical points, evaluate the function at those points to find max's and min's.
2. Take the derivative, find all the critical points, determine if the derivative changes sign around the critical points to find the relative extrema.
3. Take the derivative, find all the critical points, take the second derivative, evaluate the critical points on the second derivative to determine the concavity at the critical points and find relative extrema.

And now - The MegaProblem

Polly's no Meal's Packaging Problem

Polly's No-Meal is a popular new restaurant in the downtown Atlanta area. Though everyone enjoys their food, their patronage consists largely of one group in particular: mathematicians. It is believed that wild mathematicians are drawn to the restaurant by their name, which sounds similar to the word "polynomial." Given their hit success with the mathematician community, Polly's No-Meal has decided to cater their service directly to the tastes of mathematicians. They redesigned their menu in order to make it more mathematically oriented. All prices are given in duodecimal since the common mathematician has a tendency to emit a series of barks and growls, interspersed with theorems recited from memory, whenever confronted with a number that is not in the base numeral system of an antiprime. The entire establishment is also divided into two sides, pi and tau. This is because mathematicians have such strong preferences about these constants that if they were to be seated in the same area, chaos would ensue. All the tables in the establishment are equilateral triangles, allowing them to be tessellated together easily to form larger tables.

Due to these changes, Polly's No-Meal has become a thriving restaurant, attracting various breeds of mathematicians from all over the city. Wild mathematicians have been frequently been seen covering their tables with spreadsheets and muttering to themselves that "No! My model is still off by 2.47%." The napkins are used more often as scratch paper than as cleaning devices, and shouts of "Eureka!" can regularly be heard a quarter mile away.

Recently, the restaurant has encountered a bit of trouble. Last Monday, just as all the regular customers were slinking away to find some unaware English Major to prey on, a group of three mathematicians walked in with a menacing sway. As if on cue, all the light fixtures went out, leaving only the soft glow of the mathematicians' ears. The mathematicians slowly approached Polly, with only soft wheezing noises coming from their eyes to permeate the silence. One, which wore a large fur coat covering a neon green swimsuit and a large hat that read "Elephants? More like baby terrorists," spoke in a quiet, raspy voice. "What a nice little restaurant this is." As he spoke, low clicking noises came from the general head regions of the other two mathematicians. "My associates and I enjoyed our meal, but were thoroughly unsatisfied with the dining experience itself. You see, mortal one, we ordered the alphabetic variable soup, and the container in which it was served did not have maximized volume efficiency. This…displeases us, and nobody wants to displease us."

The first mathematician ceased speaking, and a second stepped forward. They wore a dark suit jacket over a darker black sweater-vest and a purple sequined fedora at an angle that enshrouded their face in darkness, making it difficult to discern their gender. They began to speak in that same gravelly voice, saying, “We will return in a week. We expect that by that time, you will have solved this little problem. If not, you shall face dire consequences. If we feel particularly merciful, we may just divide you by zero.” As they said this, the mathematician held up in one hand a crude effigy of Polly constructed out of wire, hay, and graph paper, and below it, in the other hand, a small zero. As they did this, the third mathematician, who wore a flowing black robe emblazoned with sacred geometry, stepped forward with a toothy smile showing her extra third row of teeth, a marked difference from the normal two rows. She held out a pointing finger, and made a sharp slashing motion. Simultaneously, a streak of flame shot in the same motion pattern between the doll and the zero, forming a fiery fraction bar. The effigy caught fire, and the mathematician in the fedora dropped it onto the floor and let it burn.

The mathematician in the robe knelt down before the burning doll and began to chant in an obscure language. The chant roughly translated to, "We bring this offering to you, oh great Newton, master of our being, bringer of our prey, and owner of our souls. Just as you defeated Leibniz in unlocking the secrets of our universe, so too shall we defeat this mortal one, whose restaurant is an affront to our core belief in the sacred power of optimization. We shall rain destruction upon her in service to you, our great and powerful ruler. All praise is your rightful prize, oh great Newton." After reciting this in about two minutes, the robed mathematician took the zero from the one in the fedora and dropped it onto the flaming effigy, causing the zero to vaporize instantly.

The mathematicians looked up from the doll. Simultaneously, they rasped, "Act wisely, mortal." With that, the three mathematicians turned to leave. As they walked out, the robed one placed a small ring on one of the triangular tables. And then they left, with their double tails swaying behind them as they walked out of Polly's No-Meal.

After the mathematicians left, Polly went to investigate the ring. It was a mobius loop, engraved with the words, "Fear the wrath of Newton," around its singular face. She turned to face the doll, still burning on the tile floor. She retrieved the fire extinguisher and sprayed it at the doll, but the fire wouldn't go out. After trying water and dark magic to remove the fire, she decided she would just have to wait it out. She stood vigil over that fire through the night. Then, as the clock struck 3:14 AM, the fire immediately went out, and the doll disintegrated into a pile of ash that smelled faintly of lavender and iron filings.

The next morning, as early mathematicians began lurking into the restaurant, Polly began working hastily on improving the design of her containers. Her soup containers have an important backstory. Back when Polly was first starting her business, she needed to buy land to build the restaurant on. Unfortunately, due to rising land prices and loan discrimination against people born at 285 feet or more above sea level, the only land she could afford was the site of a former combo chemical plant/Babies-R-Us. Due to the hazardous materials left over in the ground from the Babies-R-Us, like Cyanide, Nitroglycerin, and reverse chewing gum, the city would not legally allow her to open a restaurant there. After several weeks of searching law books and tomes of dark enchantments, Polly found just the legal loophole that she needed. If she could register her business as a packaging company, rather than a restaurant, she could legally operate on the site. She immediately began reconstructing her business model. The cheapest form of packaging she could sell was cardboard boxes, so she decided all her products would be overpriced cardboard boxes that came with a free meal inside each one. This way, she could legally sell her food while technically remaining a packaging company. To make it extra clear that her business was not a restaurant, she decided to name it Polly's No-Meal to emphasize to authorities that she was not selling meals. After making those changes, the restaurant launched without a hitch, eventually growing to the thriving business it is today.

Polly sat down in her office/nuclear fallout shelter/guest room to work out just what to do about the mathematicians' demands. She was no stranger to their shenanigans, so she knew she had better get down to it.

The cardboard boxes she uses for her soups are folded from 25cm x 40cm sheets of cardboard. To fold them, she cuts out a square of the same size from each corner of the sheet, and then folds up the edges to form a sort of cardboard bowl. What size square should Polly cut out from each corner of the cardboard to maximize the volume contained by the bowl and appease the demands of the feral mathematicians?

Solution

This is an optimization problem.

Let x = the side length of the square cut from each corner

V = lwh, V=x(25-2x)(40-2x) = 4x^3-130x^2+1000x

V' = 12x^2-260x+1000

V'(c) = 0 = 12x^2-260c+1000, 0=3c^2-65c+250, c=5 or c=50/3

V''(x) = 24x-260, V''(5) = -140, V''(50/3) = 140, therefore x=5 is a rel max of V and Polly should cut 5cm x 5cm squares.

Disclaimer

The following problem comes from the demented mind of Cody "Malfeasance" Nelson. I don't pretend to know what it means, I just do the math. Understanding the concept of discretion is advised.

Ferret out a Solution (By Cody “Malfeasance” Nelson)

Ferret Polishing Unlimited is a new company that just opened in Atlanta. Their services allow you affordably hire someone to do the grueling, back-breaking work of polishing your pet ferret. Ferret Polishing Unlimited wants to charge the cheapest price where they will always break even, but they must cost the same amount for all ferrets due to ferret anti-discrimination laws. A t-year-old ferret's surface area is $S_A(t) = -5\,t^2 + 70\,t + 75$ and ferret polish cost per square centimeter of ferret is C=.25A + 5 in dollars. How old will the most costly ferret be, and how much will it cost to polish?

Solution

This is another optimization problem. First we need to model the cost in terms of age, so we plug the surface area formula into the cost formula and get:

$C(t) = -1.25t^2 + 17.5t + 23.75$

We take the derivative and get

$C'(t) = -2.5t + 17.5$, $C'(k) = 0 = -2.5k + 17.5$, $k = 7$

$C''(t) = -2.5 < 0$, therefore $t = 7$ is a rel max of $C(t)$

$C(7) = -1.25(7)^2 + 17.5*7 + 23.75 = \$85$. A 7 year old ferret costs $85

The Emergency Meteor Dome (By Powell Whitaker)

4095 CE

On a remote planet in the outer reaches of the galaxy, basic infrastructure was set up for a new city. 2 years later, enough people had moved in that the city had grown to a size which warranted a mayor. The city elected Howard Stanley, a respected politician who had a part in the original founding of the city. However, they quickly came to find out that Stanley was a tad paranoid, made evident by his rogue meteor impact prevention policy. This policy called for the creation of a colossal dome constructed from glass nanites, which would reside outside the atmosphere in geosynchronous orbit. In the event that the city was in danger of a meteor impact, the dome would be lowered around the city to protect it. Despite the blatantly impractical nature of such an idea, the policy went into effect 2 months after Mayor Stanley took office.

The budding city continued to thrive in the time following, though the minimal amount of safety offered by the glass dome looming overhead had little to do with it, likely hindering the city's growth with its tax burden rather than helping. Even still, the population grew at an exponential rate, freed from the natural population limits by steady advances in technology. This posed a problem for the rogue meteor impact prevention policy because as the population grew, so did the size of the city, eventually surpassing the radius of the dome. To fit the expanding city limits within the protection of the dome, mayor Stanley amended the policy to include continuous expansion of the dome to match the size of the city. More nanites were to be added daily to fulfill this regulation.

This is where you come in. The legislative body of the city wants an estimate of the burden this new development will place on the residents. The growth of the city's area has been modeled as A = 200t+1200, where A is area in km^2 and t is time in years from this date. Thanks to the meticulous work put into city planning, the city limits always take the shape of a perfect circle., so the dome can fit snugly around it. Expanding the dome's surface area by 1 km2 requires 50 canisters of nanites. To get an idea of how resource intensive the expansion of the dome will be, calculate the number of canasters the expansion will require per day after 5 years of growth. (There are 160 days in a year here.) For reference, the area of a circle is πr^2, and the surface area of a hemisphere is $2\pi r^2$.

Solution

Given: dA/dt = 200, t = 5

Find: dSA/dt

Relationship: $A = \pi r^2$, $SA = 2\pi r^2$

Combine the two relationships to get SA = 2A

Differentiate: dSA/dt = 2dA/dt

dSA/dt = 2*200 = 400km^2 per year

400km^2 per year = 20,000 canisters of glass nanites per year = 125 canisters per day

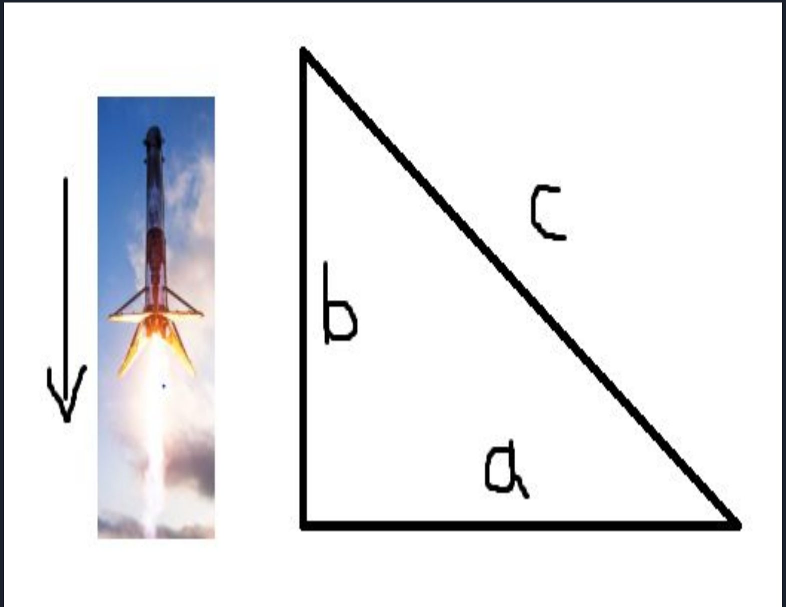
Solution Continued

Thanks to you, the city realized that the relationship between the city size and the dome size is linear, so its rate of growth remained constant. Many years later, the city did finally need the protection system due to an incoming meteor. Unfortunately, due to lack of forethought, the impact of the dome’s drop from orbit obliterated the city, doing far more damage than the meteor ever would have.

SpaceX Landing

There is a camera that is mounted 2,000 ft from the landing pad at the site of the upcoming spaceX ground landing. To keep the spacecraft in focus the camera must pivot as it comes in to land / crash. If the rocket is coming into a vertical landing at a constant 150 ft / sec, when it is 2,100ft in the air how fast is the camera-to-rocket distance changing?

solution



Given: a = 2,000 b = 2,100 c = 2,900

da/dt = 0 db/dt = -150

Find: dc/dt

Relationship: $a^2 + b^2 = c^2$

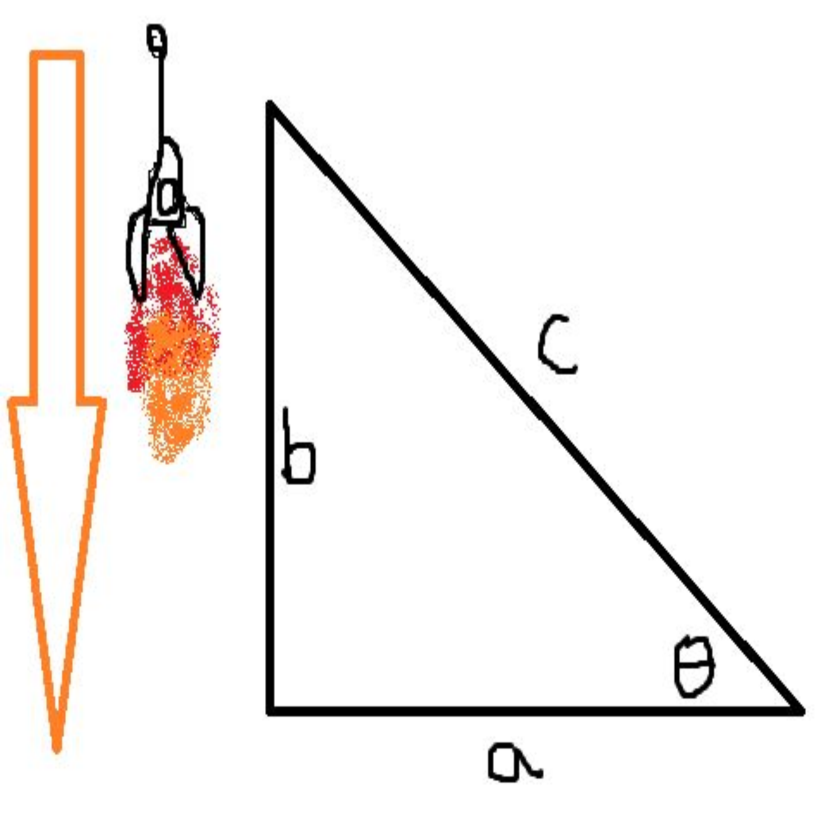
$2000^2 + b^2 = c^2$

2*b*(db/dt) = 2*c*(dc/dt)

2100*-150 = 2900*(dc/dt) dc/dt = -108.62 $m*s^{-1}$

SpaceX part b landing

How fast is the angle of elevation from the camera to the rocket changing at 2100 ft in the air?



Given: a = 2,000 b = 2,100 c = 2,900

da/dt = 0 db/dt = -150 secθ = hy / adj = c/a = 29/20

Find : dθ/dt

Relationship: tanθ = b/a

$\tan\theta = (1/2000) * b$

$\sec^2\theta * (d\theta/dt) = (1/2000) * (db/dt)$

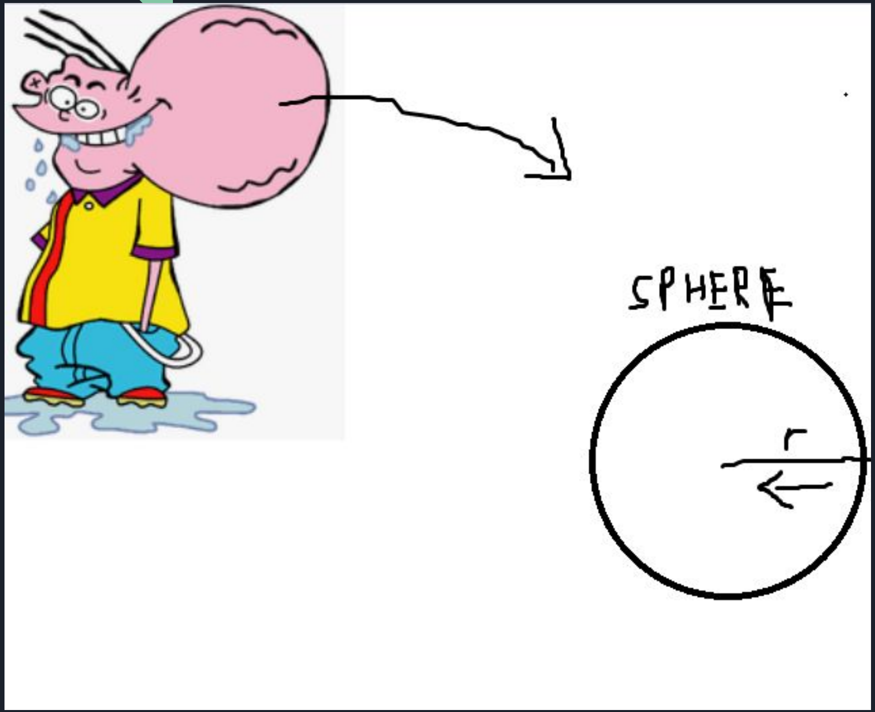
$(29/20)^2 * (d\theta/dt) = (-150/2000)$

$d\theta/dt = -30/841 = -0.03567$ radians / sec

Jawbreaker

Eddy is thoroughly enjoying his new Jawbreakers!™ jawbreaker, which he loves so much because they are paradoxically a perfect mathematical sphere. As he sucks it, its radius decreases at a rate of 21/(8*pi) cm / min. How blazingly fast is Eddy shrinking the jawbreaker when its radius is 2 cm?

The soluition



Given : r = 2cm   dr/dt = -21/(8*pi) cm per min

Find: dv/dt

Relationship $v = (4/3) * pi * r^3$

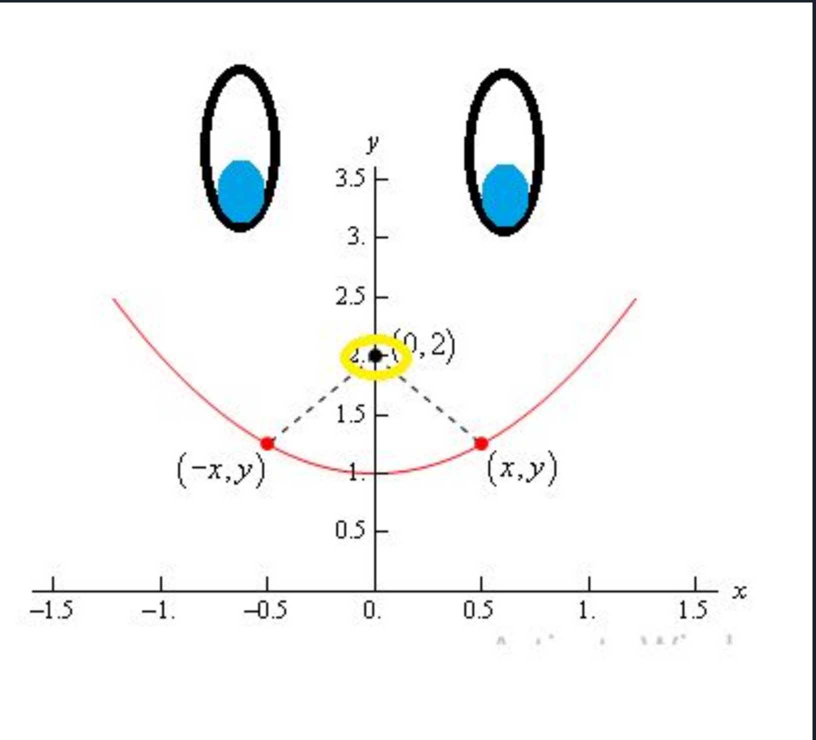
$dv/dt = (4/3) * pi * 3 * r^2 * (dr/dt)$

$dv/dt = 4 * pi * r^2 * (dr/dt)$

$dv/dt = 4*pi*2^2 * -21/(8*pi)$

$= -42\ cm^3$ per min

What points on $y = x^2 + 1$ are closest to $(0,2)$?

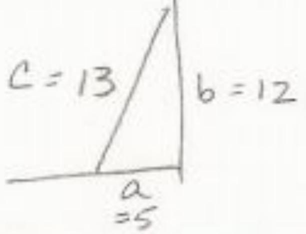


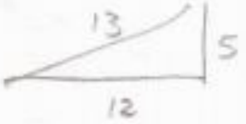
y = x^2 + 1
d = sqrt((x - 0)^2 + (y - 2)^2)
 = sqrt(x^2 + (y - 2)^2)
D = d^2 = x^2 + (y - 2)^2
 rearrange equation so x^2 = y-1
D(y) = y -1 + (y -2)^2 = y^2 - 3y + 3
D’(y) = 2y - 3
D”(y) = 2
there is a crit pt. at y = 3/2
and second deriv. is always positive so
x^2 = (3/2) - 1 = ½
x = + or - 1/sqrt(2)

points are (1/sqrt(2), 3/2) and (-1/sqrt(2), 3/2)

And now, regular problems (ugh)

Example 6) A 13 foot ladder leans against a vertical wall. If the lower end of the ladder is pulled away at the rate 2 feet per second, how fast is the top of the ladder coming down the wall at a) the instant the top is 12 feet above the ground and b) 5 feet above the ground?





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a) the instant the top is 12 feet above the ground and

Given: $b = 12$, $\frac{da}{dt} = 2$ ft/sec, $a = 5$

Find: $\frac{db}{dt} = ?$

Rel: $a^2 + b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$

$(5)(2) + (12)\left(\frac{db}{dt}\right) = 0 \rightarrow \frac{db}{dt} = \frac{-5}{6}$ ft/sec

b) 5 feet above the ground?

Given: $b = 5$, $\frac{da}{dt} = 2$ ft/sec, $a = 12$ ft

Find: db/dt

Rel: $a^2 + b^2 = 13^2$

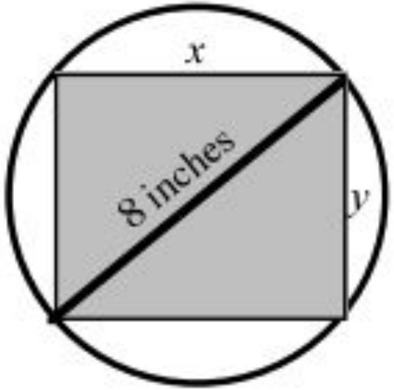
$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$

$(12)(2) + (5)\left(\frac{db}{dt}\right) = 0$

$5\frac{db}{dt} = -24 \rightarrow \frac{db}{dt} = \frac{-24}{5}$ ft/sec

Example 6) . Find the dimensions of the largest area rectangle which can be inscribed into a circle of radius 4 inches.

primary *secondary*



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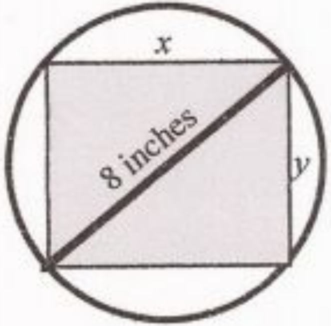


primary

$R = x \cdot y$

$R = x(64 - x^2)^{1/2}$

$R' = \frac{x}{2}(64 - x^2)^{-1/2}(-2x) + (64 - x^2)^{1/2}$

$R' = \frac{-x^2}{\sqrt{64 - x^2}} + \frac{\sqrt{64-x^2}\,\sqrt{64-x^2}}{\sqrt{64-x^2}}$

secondary

$x^2 + y^2 = 8^2$

$y^2 = 64 - x^2$

$y = \sqrt{64 - x^2}$

$\rightarrow R' = \frac{64 - 2x^2}{\sqrt{64 - x^2}} = 0$

$\Rightarrow 64 = 2x^2$ or R' DNE $\Rightarrow 64 = x^2$

$32 = x^2$ $8 = x$

$x = \pm 4\sqrt{2}$

R:

R′: + on [0, $4\sqrt{2}$], − on [$4\sqrt{2}$, 8]

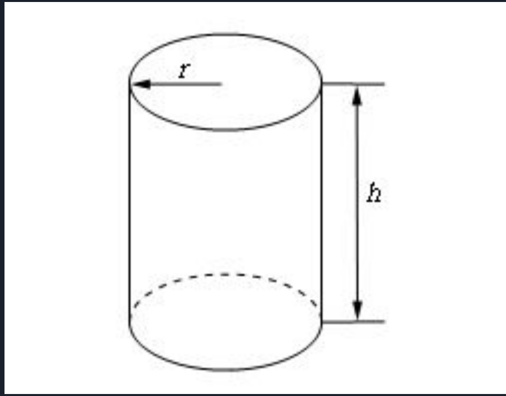
R has a max if $x = 4\sqrt{2}$ + $y = 4\sqrt{2}$

How would this problem change if the radius were *r* inches?

The secondary equation would be $y = \sqrt{r^2 - x^2}$

Example 7) A 6 oz. can of Friskies Cat food contains a volume of approximately 14.5 cubic inches. How should

A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can that will minimize the amount of material used in its construction.



$V=(\pi r^2)(h)=\pi r^2 h \quad A=(2\pi r)(h)=2\pi rh$

Minimize : $A=2\pi rh+2\pi r^2$
Constraint : $1500=\pi r^2 h$

$h=1500/\pi r^2 \Rightarrow A(r)=2\pi r(1500/(\pi r^2)) + 2\pi r^2$
$=2\pi r^2+3000/r$

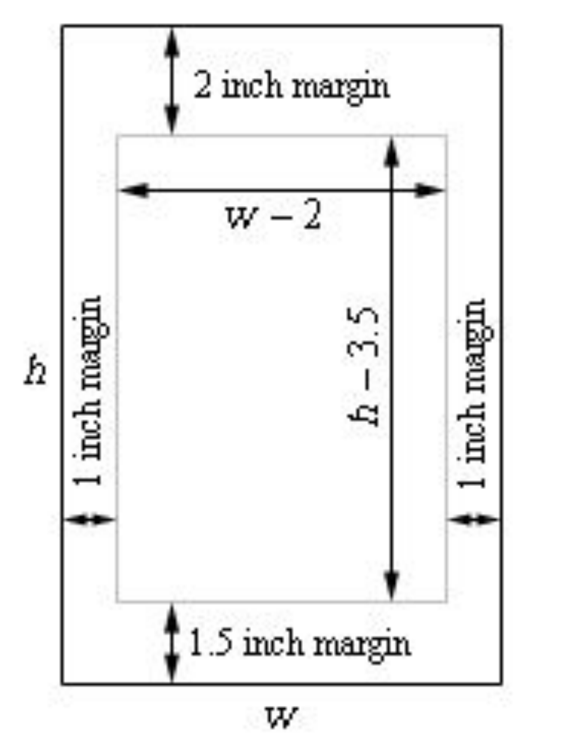
$A'(r)=4\pi r-3000/r^2=(4\pi r^3-3000)/r^2$

crit point at $r = 6.2035$

$h=1500 / (\pi(6.2035)^2) = 12.4070$

Therefore, if the manufacturer makes the can with a radius of 6.2035 cm and a height of 12.4070 cm the least amount of material will be used to make the can.

A printer need to make a poster that will have a total area of 200 in^2 and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom as shown below. What dimensions will give the largest printed area?



$$A = (w-2)(h-3.5)$$
$$200 = wh$$

$$A(w) = (w-2)\left(\frac{200}{w} - 3.5\right) = 207 - 3.5w - \frac{400}{w}$$

$$A'(w) = -3.5 + \frac{400}{w^2} = \frac{400 - 3.5w^2}{w^2} \qquad A''(w) = -\frac{800}{w^3}$$

$$w = \pm\sqrt{\frac{400}{3.5}} = \pm 10.6904$$

$$h = \frac{200}{10.6904} = 18.7084 \text{ inches}$$

Air is being pumped into a spherical balloon at a rate of 5 cm^3/min. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm.

$$V'(t) = 5$$

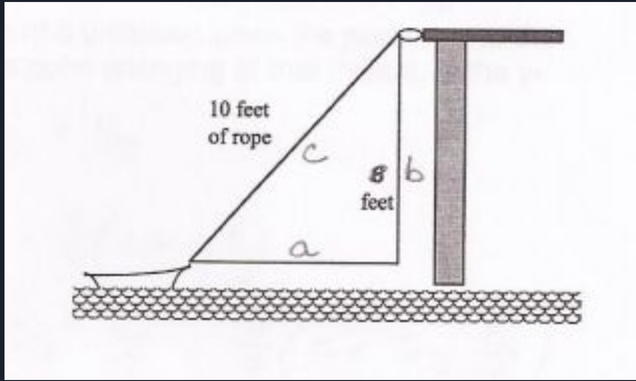
$$r'(t) = ? \quad \text{when} \quad r(t) = \frac{d}{2} = 10\,\text{cm}$$

$$V(t) = \frac{4}{3}\pi[r(t)]^3$$

$$V' = 4\pi r^2 r'$$

$$5 = 4\pi\left(10^2\right)r' \qquad \Rightarrow \qquad r' = \frac{1}{80\pi}\ \text{cm/min}$$

23. A rowboat is pulled toward a dock from the bow through a ring on the dock 8 feet above the bow. If the rope is hauled in at 3 ft/sec, how fast is the boat approaching the dock when 10 feet of rope are out?



Given: $c = 10$, $b = 8$, $a = 6$ $\left(\overset{b/c}{3,4,5}\right)$

$$\frac{dc}{dt} = 3 \text{ ft/sec}, \quad \frac{db}{dt} = 0$$

Find: $\frac{da}{dt} = ?$

Rel. $a^2 + b^2 = c^2$

$$a^2 + 8^2 = c^2$$

$$2a \frac{da}{dt} = 2c \frac{dc}{dt}$$

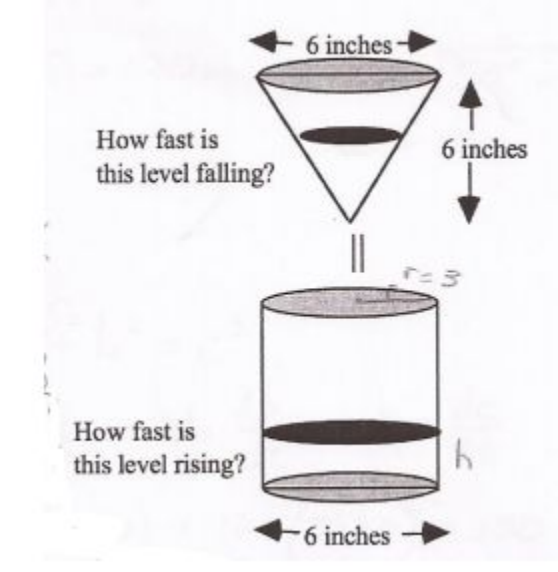
$$6 \frac{da}{dt} = 10(3)$$

$$\boxed{\frac{da}{dt} = 5 \text{ ft/sec}}$$

26. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 10 in^3/min.

a) How fast is the level in the pot rising when the coffee in the filter is 5 inches deep?

b) How fast is the level in the cone falling then?



Given: $\frac{dV}{dt} = 10 \frac{in^3}{min}$, $h = 5$, $r = 3$, $\frac{dr}{dt} = 0$ } cylinder

Find: $\frac{dh}{dt} = ?$

Rel: $V = \pi r^2 h$

$V = 9\pi h$

$\frac{dV}{dt} = 9\pi \frac{dh}{dt}$

$\frac{10}{9\pi} = \frac{dh}{dt}$

cone!

b) Given $\frac{dV}{dt} = -10 \frac{in^3}{min}$

$h = 5$, $2r = h \Rightarrow r = \frac{h}{2}$

Find: $\frac{dh}{dt}$

Rel: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 \cdot h$

$V = \frac{\pi}{12} \cdot h^3$

6 inches

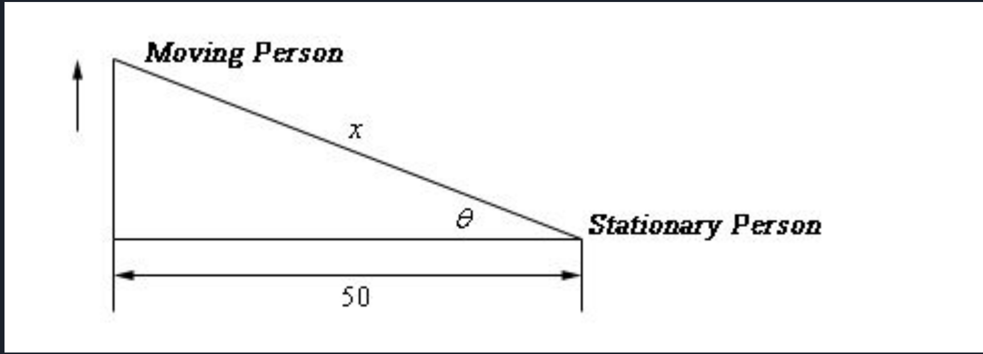
$\frac{dV}{dt} = \frac{3\pi}{12} \cdot h^2 \cdot \frac{dh}{dt}$

$-10 = \frac{\pi}{4} \cdot 5^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = -10 \cdot \frac{4}{\pi} \cdot \frac{1}{25} = \frac{-8}{5\pi}$

Two people are 50 feet apart. One of them starts walking north at a rate so that the angle shown in the diagram below is changing at a constant rate of 0.01 rad/min. At what rate is distance between the two people changing when $\theta=0.5$radians?

rate of 0.01 rad/min.



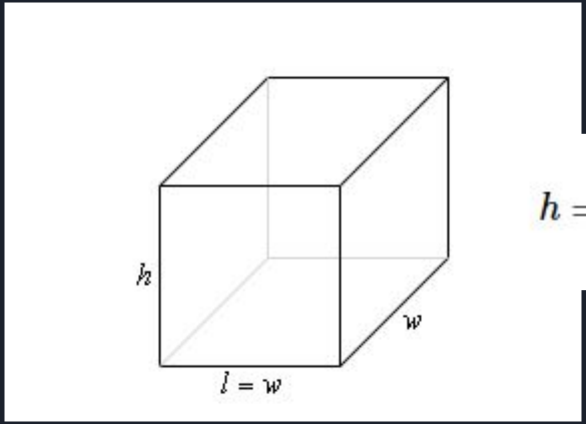
$$\cos\theta = \frac{50}{x} \qquad \sec\theta = \frac{x}{50}$$

$$\sec\theta\tan\theta\,\theta' = \frac{x'}{50}$$

$$(50)(0.01)\sec(0.5)\tan(0.5) = x' \quad \Rightarrow \quad x' = 0.311254 \text{ ft}/\min$$

We want to construct a box with a square base and we only have 10 m^2 of material to use in construction of the box. Assuming that all the material is used in the construction process determine the maximum volume that the box can have.

$$\text{Maximize}: V = lwh = w^2h$$
$$\text{Constraint}: 10 = 2lw + 2wh + 2lh = 2w^2 + 4wh$$



$$h = \frac{10 - 2w^2}{4w} = \frac{5 - w^2}{2w} \quad \Rightarrow \quad V(w) = w^2\left(\frac{5 - w^2}{2w}\right) = \frac{1}{2}\left(5w - w^3\right)$$

$$V'(w) = \frac{1}{2}\left(5 - 3w^2\right) \qquad V''(w) = -3w$$

$$w = \pm\sqrt{\frac{5}{3}} = \pm 1.2910$$

$$V(1.2910) = 2.1517\,\text{m}^3$$

$$l = w = 1.2910 \qquad h = \frac{5 - 1.2910^2}{2(1.2910)} = 1.2910$$

A window is being built and the bottom is a rectangle and the top is a semicircle. If there is 12 meters of framing materials what must the dimensions of the window be to let in the most light?

$$\text{Maximize}: A = 2hr + \frac{1}{2}\pi r^2$$

$$\text{Constraint}: 12 = 2h + 2r + \pi r$$

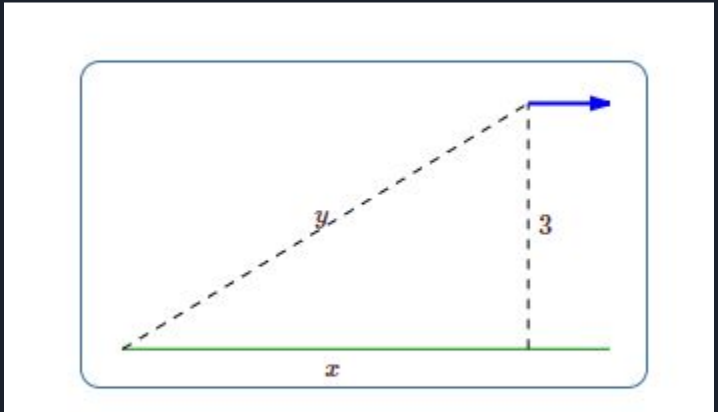
$$h = 6 - r - \frac{1}{2}\pi r \qquad \Rightarrow \qquad A(r) = 2r\left(6 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}\pi r^2 = 12r - 2r^2 - \frac{1}{2}\pi r^2$$

$$A'(r) = 12 - r(4+\pi) \qquad A''(r) = -4 - \pi$$

$$r = \frac{12}{4+\pi} = 1.6803$$

We can also see that the second derivative is always negative (in fact it’s a constant) and so we can see that the maximum area must occur at this point. So, for the maximum area the semicircle on top must have a radius of 1.6803 and the rectangle must have the dimensions 3.3606 x 1.6803 (h x 2r).

A plane is flying directly away from you at 500 mph at an altitude of 3 miles. How fast is the plane's distance from you increasing at the moment when the plane is flying over a point on the ground 4 miles from you?



, $dx/dt = 500$.

$x^2 + 9 = y^2$

$$2x\dot{x} = 2y\dot{y}.$$

We are interested in the time at which $x = 4$; at this time we know that $4^2 + 9 = y^2$, so $y = 5$. Putting together all the information we get

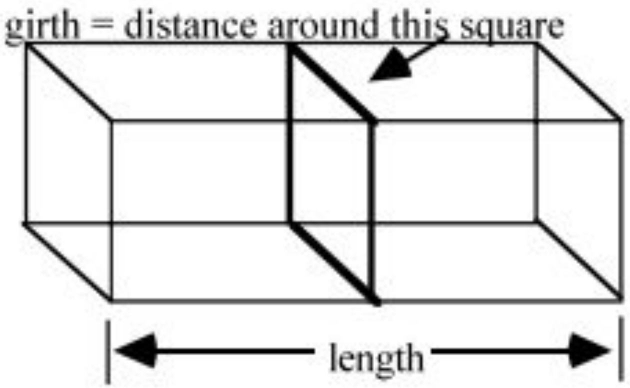
$$2(4)(500) = 2(5)\dot{y}.$$

Thus, $\dot{y} = 400$ mph.

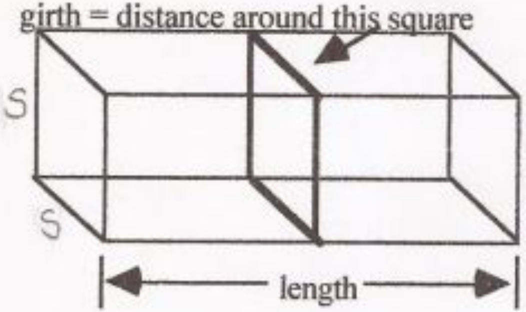
You are inflating a spherical balloon at the rate of 7 cm33/sec. How fast is its radius increasing when the radius is 4 cm?

Here the variables are the radius rr and the volume V. We know dV/dt, and we want dr/dt. The two variables are related by means of the equation $V=4\pi r^3/3$. Taking the derivative of both sides gives $dV/dt=4\pi r^2 r'$. We now substitute the values we know at the instant in question: $7=4\pi 4^2 r'$, so $r'=7/(64\pi) r'=7/(64\pi)$ cm/sec.

7. The U.S. Postal Service will accept a box for domestic shipping only if the sum of the length and the girth (distance around) does not exceed 108 inches. Find the dimensions of the largest volume box with a square end that can be sent.



7. The U.S. Postal Service will accept a box for domestic shipping only if the sum of the length and the girth (distance around) does not exceed 108 inches. Find the dimensions of the largest volume box with a square end that can be sent.



$108 = 4s + l$

$V = s^2 l$

$V = s^2(108 - 4s)$

$V = 108s^2 - 4s^3$

$V' = 216s - 12s^2 = 0$

$V' = 12s(18 - s) = 0$

$s = 0$ or $s = 18$

V': $-$ | 0 | $+$ | 18 | $-$

Dimensions of largest box w/ square end are

$18 \times 18 \times 36$

Fin